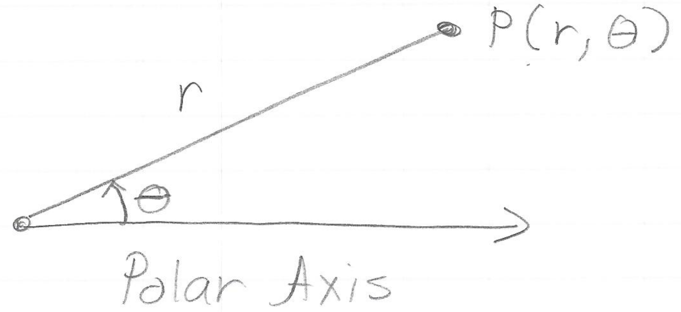
Guided Notes

Section 10.3

Polar Coordinates

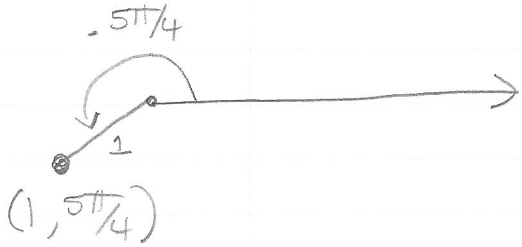


$x = r\cos\theta$

$y = r\sin\theta$

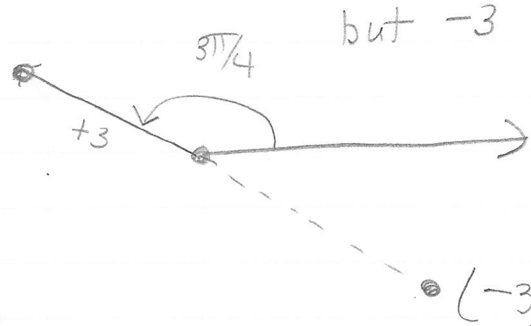
Example | Plot $(1, 5\pi/4)$

$\frac{5\pi}{4} = 1\frac{1}{4}\pi$



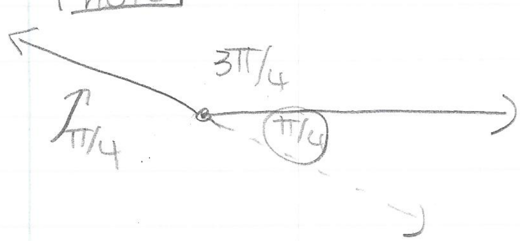
Plot $(-3, 3\pi/4)$

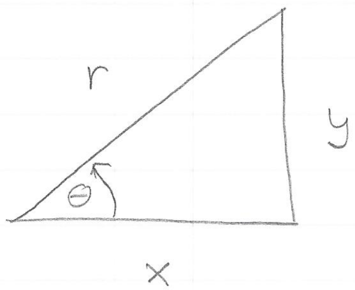
but -3 changes things.



$= (3, 7\pi/4)$

note





$x = r\cos\theta$
$y = r\sin\theta$

$r^2 = x^2 + y^2$

$\tan\theta = \dfrac{y}{x}$

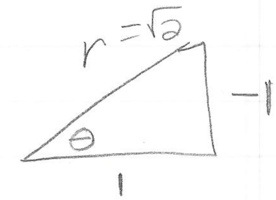
$\boxed{\text{Example 1}}$ Convert the cartesean coordinates $(1,-1)$ into polar coordinates.

Recall: $x = r\cos\theta$ $\quad x^2 + y^2 = r^2$.
$y = r\sin\theta$

$(+1)^2 + (-1)^2 = r^2$
$2 = r^2$
$\boxed{\sqrt{2} = r}$

$\tan\theta = \dfrac{y}{x}$

$\tan\theta = \dfrac{-1}{1}$



(not oriented properly)

(r, θ)
So there are many possible answers

$\tan^{-1}(-1) = \theta$

$\theta = -\dfrac{\pi}{4}, \dfrac{7\pi}{4}, \ldots$

2 are

$\boxed{\left(\sqrt{2}, \frac{-\pi}{4}\right) \quad \left(\sqrt{2}, \frac{7\pi}{4}\right)}$

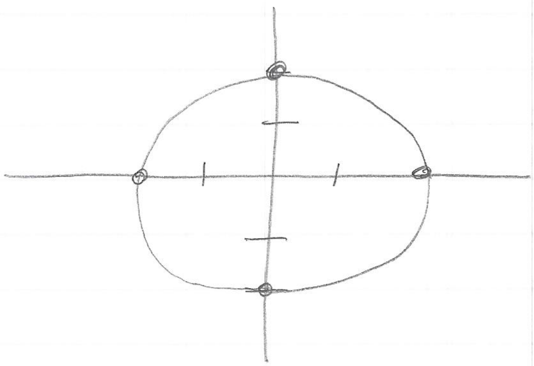
$r = f(\theta)$ | $F(r, \theta) = 0$

Polar Curves

Example 2 $r = 2$

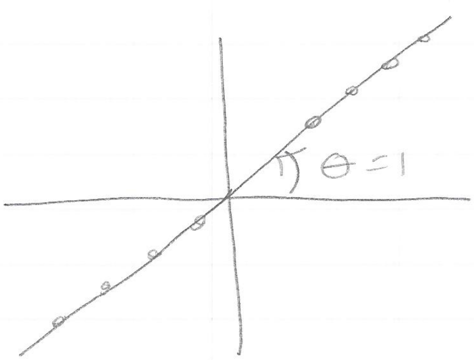
so $r=2$ always (for all θ)

$\Rightarrow$ circle



Example 3 $\theta = 1$

the angle is alway 1 radian



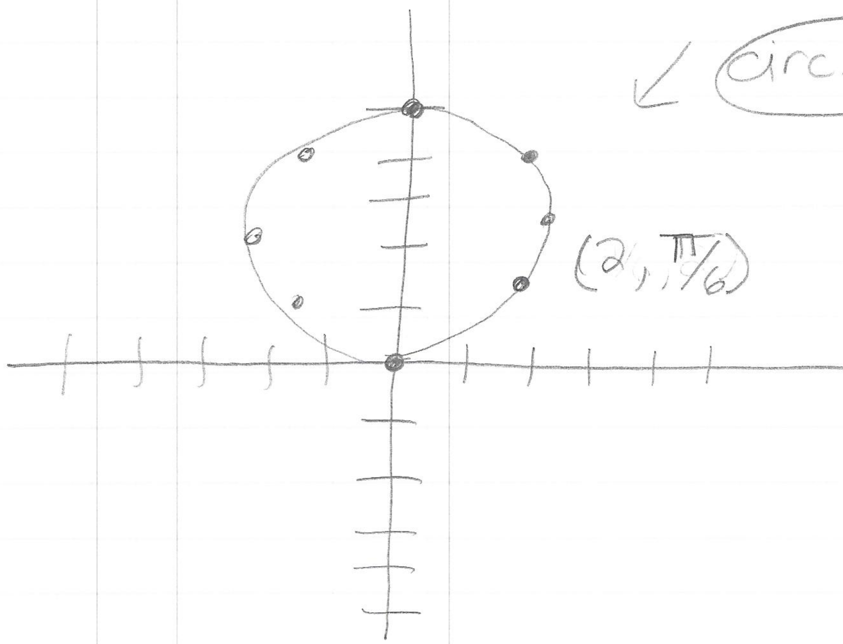
Example 4 $r = 4\sin\theta$

make an r, θ table. Then plot the points.

θ	r
0	0
$\pi/6$	2
$\pi/4$	$2\sqrt{2}$
$\pi/3$	$2\sqrt{3}$
$\pi/2$	4
$2\pi/3$	$2\sqrt{3}$
$3\pi/4$	$2\sqrt{2}$
$5\pi/6$	2
π	0

stop when function begins to repeat

circle



$\boxed{\text{Example 5}}$ $r = 2 + 2\cos\theta$

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	$5\pi/6$	π
r	4	$2+\sqrt{3}$	$2+\sqrt{2}$	3	2	1	$2-\sqrt{2}$	$2-\sqrt{3}$	0

Polar graph paper helps.

$\boxed{\text{cardioid}}$ (heart)

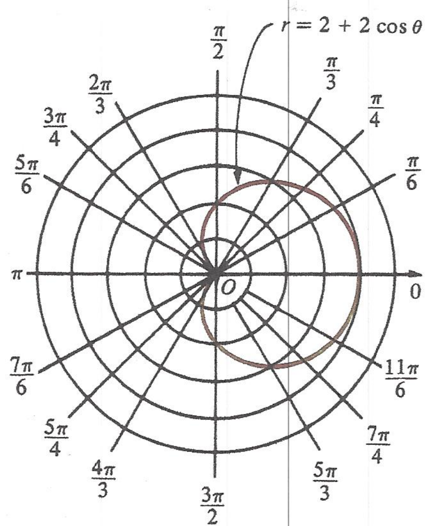


FIGURE 13.10

$r = a(1+\cos\theta)$
$r = a(1-\cos\theta)$

$r = a(1+\sin\theta)$
$r = a(1-\sin\theta)$

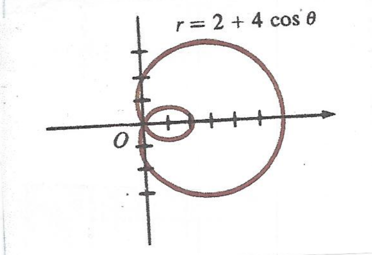
general form
$r = a \pm b\cos\theta$

$r = a \pm b\sin\theta$

$\boxed{\text{limacon}}$

(can have a loop)

Example 6 $r = 2 + 4\cos\theta$



r	θ
0	6
$\pi/6$	$2+2\sqrt{3}$
$\pi/4$	$2+2\sqrt{2}$
$\pi/3$	4
$\pi/2$	2
$2\pi/3$	0
$3\pi/4$	$2-2\sqrt{2}$
$5\pi/6$	$2-2\sqrt{3}$
π	-2

use calculator to find decimal approximations.

Example 7 Sketch the graph of the equation $r = a\sin 2\theta$, $a > 0$

$\sin 2\theta \rightarrow$ 4 leafed rose

Recall period of $\sin 2\theta = \pi$

normally the high/low points for $\sin x$ are

$x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$

(0) (1) (0) (−1) (0)

For $2\theta \longrightarrow$

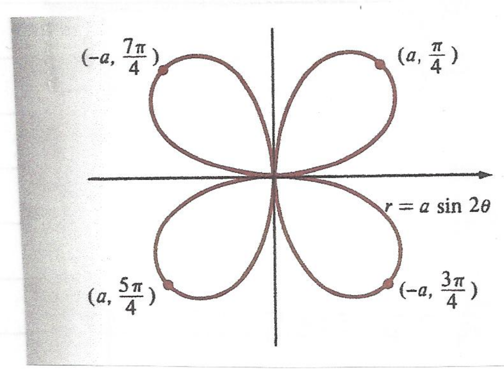
$0 \rightarrow 0$

$\boxed{\pi/4 \rightarrow 1(a)}$

$\pi/2 \rightarrow 0$

$\boxed{3\pi/4 \rightarrow -1(a)}$

$\pi \rightarrow 0$



we will need to also include $[\pi, 2\pi]$ to get all 4 petals of the rose.

on left side

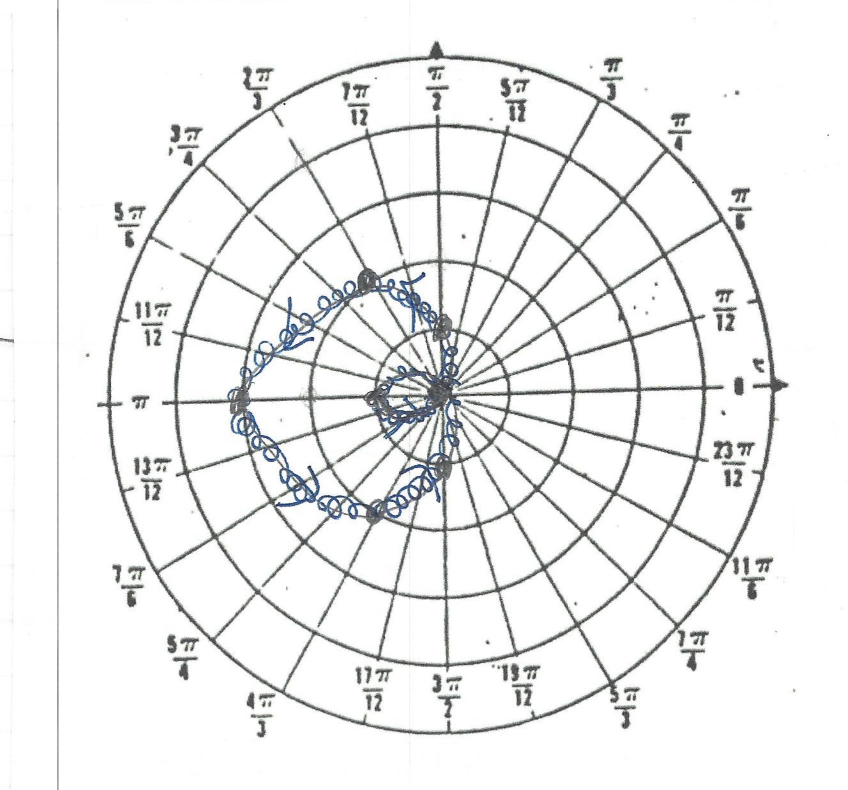
Example 8 $\boxed{\text{Graph } r = 2 - 4\cos\theta}$

recall $r = a \pm b\cos\theta$ with $a = 2$
$b = 4$

$\frac{a}{b} = \frac{2}{4} < 1 \Rightarrow$ $\boxed{\text{limacon with inner loop}}$

<u>also recall</u> positive angles (θ) $\rightarrow$ counter clockwise
negative angles (θ) $\rightarrow$ clockwise

r	θ
-2	0
0	$\pi/3$
2	$\pi/2$
4	$2\pi/3$
6	π
$\vdots$	$\vdots$



scale
each ring $= 2$

Example 9

Sketch a graph of
$r = 4\cos 2\theta$ and $r = 2$
Find all intersection points

$r = 4\cos 2\theta$
(4 petals)

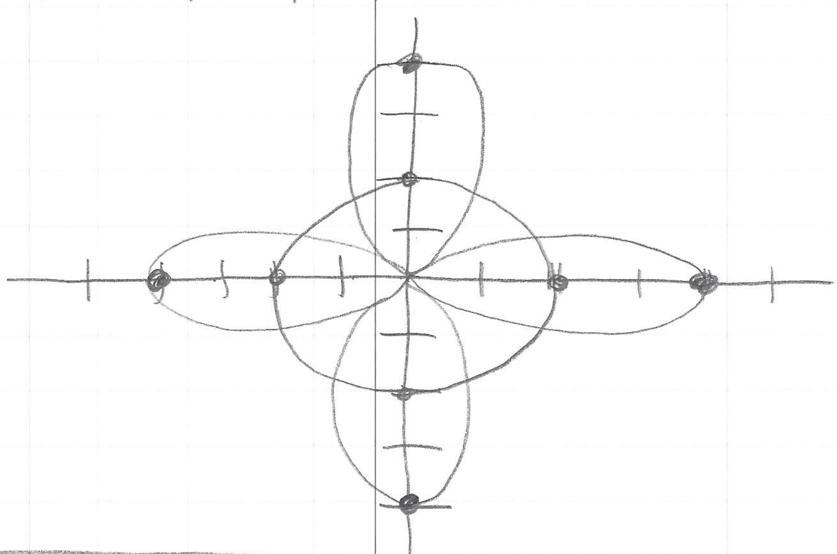
$r = 2$
circle
(radius 2)

$0 \le \theta \le 2\pi$
$0 \le 2\theta \le 4\pi$

The graph

$r = 4\cos 2\theta$ tips of petals for $\cos\theta$ $\rightarrow 0, \pi, 2\pi, 3\pi, 4\pi, \ldots$

$2\theta = 0, \pi, 2\pi, 3\pi, 4\pi, 5\pi, \ldots$
$\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi, \frac{5\pi}{2}, 3\pi, \ldots$
$r = 4 \quad -4 \quad 4 \quad -4 \quad 4 \quad -4 \quad 4 \ldots$



$r=2$
circle
$r=2$

8 intersection points

The intersection points

$2 = 4\cos 2\theta$

$\frac{1}{2} = \cos 2\theta$

$2\theta = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}$ $\boxed{r=2}$

$\boxed{\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}}$

also $\boxed{\cos 2\theta = -\frac{1}{2}}$

$2\theta = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}$

$\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$
$(r = -2)$

Example 10

For $r = 1 + \sin\theta$
Find the slope of the tangent line when $\theta = \frac{\pi}{3}$

$r = 1 + \sin\theta$

Recall $x = r\cos\theta$
$y = r\sin\theta$
for polar coordinates

$\frac{dr}{d\theta} = \cos\theta$

$$\frac{dy}{dx} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

$$\frac{dy}{dx} = \frac{\cos\theta\sin\theta + (1+\sin\theta)\cos\theta}{\cos\theta\cos\theta - (1+\sin\theta)\sin\theta}$$

$$\boxed{m_{\tan}\ \theta = \frac{\pi}{3}} = \frac{\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(1 + \frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)}{\left(\frac{1}{2}\right)^2 - \left(1 + \frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)} = \boxed{-1}$$

When is the tangent horizontal?

When the numerator of the fraction $= 0$

$\cos\theta(1 + 2\sin\theta) = 0$

solve each separately to get

$$\boxed{\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}}$$